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Quantifying extreme risks in stock markets: A case of former Yugoslavian states*[1]

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Abstract

One of the reasons why investors were not prepared for heavy losses in the stock markets that occurred after the beginning of sub-prime mortgage crisis in the US lies in the curious fact that many practitioners were led to believe that there are so many independent agents participating in the stock markets that surely they must act according to Central limit theorem i.e. according to Gaussian distribution. As it turns out the paradigm of normality has let us down once again and reputation of VaR based risk measurement is seriously damaged. An alternative measure that looks very strong at these dire times and quantifies the losses that might be encountered in the tail is the conditional VaR (CVaR). While VaR represents a loss one expects at a determined confidence level for a given holding period, CVaR is the loss one expects, provided that the loss is equal to or greater than VaR. In this paper the testing of CVaR models is performed on stock indexes from Slovenia, Croatia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia. Error statistics show that CVaR models are quite successful at capturing extreme losses that occurred in these markets, especially models based on Generalized extreme value distribution and a proposed Hybrid historical simulation CVaR model.

Key words: *Extreme losses, Conditional VaR, Extreme value theory, Hybrid historical simulation*

JEL classification: *G24, C14, C22, C52, C53*

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1. Introduction

With the latest market crash in stock exchanges all around the world stemming from US sub-prime mortgage crises and spilling over to local stock markets it is beginning to become clear to everyone that there is a need for a risk management approach that comes to terms with the problems posed by extreme events. Risk measurement models, especially Value at risk (VaR) models that are used today in the world have clearly failed their purpose and the harsh reality caught most of the investors asleep and unprepared for such events. The surprise and disbelief was that much greater since almost all of the markets have enjoyed a prolonged period of constant growth and at first glance there was nothing in VaR forecasts that would warn the investors about such huge losses. One of the reasons why most of the investors were not prepared for such events definitely lies in the curious fact that many practitioners were led to believe that stock markets behave rationally and have so many independent agents participating in them that surely they must act according to Central limit theorem i.e. according to Gaussian distribution. This unrealistic, but seductive claim was heavily endorsed by JP Morgan's RiskMetrics VaR model which became almost a standard in the risk management industry. As it turns out the paradigm of normality has let us down once again and reputation of VaR models is seriously damaged. Artzner et al. (1997, 1999) used an axiomatic approach to the problem of defining a satisfactory risk measure. They defined attributes that any good risk measure should satisfy. They called risk measures that satisfy these axioms "coherent". It turned out that VaR is not a coherent risk measure because it does not necessarily satisfy the sub-additivity condition set out by Artzner et al. (1999). VaR can only be made sub-additive if a usually implausible assumption is imposed of returns being normally (or more generally, elliptically) distributed. Even though VaR's theoretical flaws outweigh its practical advantages, VaR has become a regulatory obligation banks have to calculate in order to construct adequate capital requirements. A very serious shortcoming of VaR is that it provides no handle on the extent of the losses that might be suffered beyond a certain threshold confidence level. VaR is incapable of distinguishing between situations where losses in the tail are only a bit worse, and those where they are overwhelming. It provides the lowest bound for losses in the tail of the loss distribution and has a bias toward optimism instead of the conservatism that ought to prevail in risk management. An alternative measure that does quantify the losses that might be encountered in the tail is the conditional VaR (CVaR). While VaR represents a loss one expects at a determined confidence level for a given holding period, CVaR is the loss one expects to suffer, provided that the loss is equal to or greater than VaR. CVaR is a coherent measure of risk in the sense of Artzner et al. (1997, 1999), while VaR is not a coherent risk measure because it does not fulfill the axiom of subadditivity. This property expresses the fact that a portfolio made of subportfolios will risk an amount which is at most the sum of the separate amounts risked by its subportfolios. However, CVaR also has its' own flaws and at present it is still not required by the regulators as a risk measure that can be used to calculate economic capital. The field of CVaR estimation and model comparison is just beginning to develop and there is an obvious lack of empirical research. After all, these two risk measures are inherently connected in the sense that from the VaR surface of the tail CVaR figures can easily be calculated. We are only beginning to investigate this new and exciting area in risk management science. Advances that have been made in VaR should not be lost with the probable (and well deserved) adoption of coherent risk measures into regulatory framework. Superior quality of VaR calculation techniques should yield superior CVaR forecasts. CVaR estimations can be significantly improved by using the knowledge obtained from advances in VaR estimation.

Since CVaR is a relatively new risk measure there are very few papers dealing with empirical testing of CVaR models. The goal of this paper is to extend the advances that have been made in VaR measurement techniques to CVaR estimation. Contributions of this paper are the following: An empirical investigation into relatively uncharted waters of tail risk assessment based on stock indexes from Slovenia, Croatia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia. A new semi-parametric approach to estimating CVaR is developed, called Hybrid historical simulation CVaR which is based on bootstrapping from a series of volatility updated tail losses. The goal of the paper is to find a CVaR model that gives the best approximation to tail losses i.e. minimizes the deviation of CVaR forecasts from actual extreme losses. The rest of the paper is organized as follows: In section 2 a review of the literature on CVaR estimation and model comparison is presented. Section 3 introduces the concept of coherent risk measures, extreme value theory and a measure of average expected loss in the tail (CVaR). The consequences of coherence are discussed and strong points of CVaR are presented. Section 3 also presents a new approach to measuring CVaR. Section 4 outlines the methodology used in the testing of CVaR models. Findings and backtesting results are also presented in this section. Section 5 concludes.

2. Literature review

Although VaR is useful for financial institutions to see the glimpse of the risks they face, an ever growing number of research papers clearly show that VaR is not an adequate risk measure. As a result, more general convex measures of risk have been proposed. Among them, Conditional VaR (CVaR) became the most popular alternative to VaR. Unlike the literature about VaR model comparison which is extensive, studies that compare CVaR model performance is extremely rare, especially papers dealing with empirical comparison. Artzner et al. (1999) showed that VaR is not necessarily sub-additive, i.e., the VaR of a portfolio may be greater than the sum of individual VaRs and therefore, managing risk by using it may fail to automatically stimulate diversification. Moreover, it does not indicate the size of the potential loss, given that this loss exceeds the VaR. To remedy these shortcomings, Artzner et al. (1997) introduced the Expected Shortfall risk measure, which equals the expected value of the loss, given that a VaR violation occurred. Basak, Shapiro (2001) suggested an alternative risk management procedure, namely limited expected losses based risk management (LEL-RM), that focuses on the expected loss also when (and if) losses occur. They substantiated that the proposed procedure generates losses lower than what VaR based risk management techniques generate. CVaR turned out to be the most attractive coherent risk measure and has been studied by a number of authors (see Acerbi et al. 2001 and Inui, Kijima, 2005). Gilli, Kellezi (2003) advocate the use of Extreme value theory in tail risk estimation due to its firm theoretical grounds to compute both VaR and CVaR estimation. Yamai and Yoshiba (2005a, b) compared the two measures—VaR and CVaR —and argued that VaR is not reliable during market turmoil as it can mislead rational investors, whereas CVaR can be a better choice overall. However, they pointed out that gains on efficient management by using the CVaR measure are substantial whenever its estimation is accurate. In other cases, they advise the market practitioners to combine the two measures for best results. Kondor, Varga-Haszonits (2008) find that whenever there is an asset in a portfolio that dominates over others in a given sample the portfolio cannot be optimized under any coherent measure on that sample, including CVaR, which leads to unbounded positions, meaning that both VaR and CVaR can sometimes face similar problems. Harmantzis, Miao, Chien (2006) test several VaR and CVaR models on S&P500, DAX, CAC, Nikkei, TSE, and FTSE indexes, as well as several currencies (USD vs. EUR, JPY, GBP and CAD). They find that for CVaR estimation, the historical method and extreme value based POT method give more correct estimations. In their study Gaussian models underestimates CVaR, while models based on Stable Paretian distribution overestimates CVaR. Angelidis, Degiannakis (2007) test the performance of various VaR and CVaR model on S&P500 index, Gold Bullion $ per Troy Ounce and US dollar/British pound exchange rate. In their paper they actually tested the impact of different volatility forecasting models within a strictly parametric framework so their results are not comparable with the results of this study. They find that different volatility models are "optimal" for different assets. One can conclude that although CVaR is a superior risk measure it lacks the depth of the theoretical and empirical research that VaR measure is not lacking. Investigation into the theoretical properties of CVaR is still in its early stages, and empirical investigation is only beginning. This paper tries to fill the gap that exists in CVaR empirical literature. Furthermore, it analyses the performance of different CVaR models on a rarely analyzed markets in general, that is Slovenia, Croatia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia.

3. CVaR and coherence

Recently, VaR as a risk measure is open to criticism from many directions. Hoppe (1999) argues that the underlying statistical assumptions are violated because they can not capture many features of the financial markets such as intelligent agents. Artzner et al. (1997, 1999) have used an axiomatic approach to the problem of defining a satisfactory risk measure. They defined attributes that a good risk measure should satisfy, and call risk measures that satisfy these axioms "coherent". A coherent risk measure ρ assigns to each loss X a risk measure $\rho(X)$ such that the following conditions are satisfied (Artzner et al. 1999):

$$\rho(tX) = t\rho(X) \qquad \text{(homogeneity)} \qquad (1)$$

$$\rho(X) \geq \rho(Y), \text{ if } X \leq Y \qquad \text{(monotonicity)} \qquad (2)$$

$$\rho(X + n) = \rho(X) - n \qquad \text{(risk-free condition)} \qquad (3)$$

$$\rho(X) + \rho(Y) \leq \rho(X + Y) \qquad \text{(sub-additivity)} \qquad (4)$$

for any number n and positive number t. These conditions guarantee that the risk function is convex, which in turn corresponds to risk aversion. That is:

$$\rho(tX + (1 - t)Y) \leq t\rho(X) + (1 - t)\rho(Y) \qquad (5)$$

VaR is not a coherent risk measure because it does not necessarily satisfy the sub-additivity condition. VaR can only be made sub-additive if a usually implausible assumption is imposed of returns being normally (or slightly more generally, elliptically) distributed (Artzner et al. 1999). Subadditivity expresses the fact that a portfolio made of subportfolios will risk an amount which is at most the sum of the separate amounts risked by its subportfolios. This is maybe the most characterizing feature of a coherent risk measure, something which belongs to everybody's concept of risk. The global risk of a portfolio will be the sum of the risks of its parts only in the case when the latter can be triggered by concurrent events, namely if the sources of these risks may conspire to act altogether. In all other cases, the global risk of the portfolio will be strictly less than the sum of its partial risks thanks to risk diversification. This axiom captures the essence of how a good risk measure should behave under the composition/addition of portfolios. It is the key test for checking whether a measurement of a portfolio's risk is consistent with those of its subportfolios. For a sub-additive measure, which CVaR is, portfolio diversification always leads to risk reduction, while for measures which violate this axiom, such as VaR, diversification may produce an increase in their value even when partial risks are triggered by mutually exclusive events. Sub-additivity is not some trivial academic invention but a crucial part of any risk measure for a number of reasons:

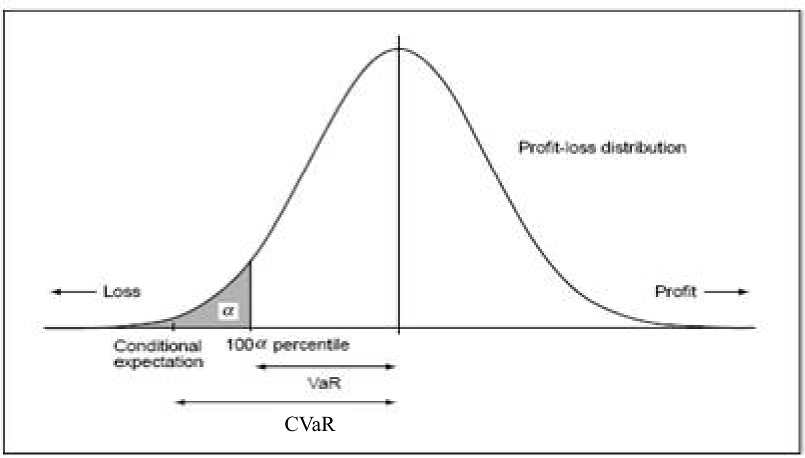
- If risks are sub-additive, then adding risks together would give an overestimate of combined risk, and this means that a sum of risks can be used as a conservative estimate of combined risk. This facilitates decentralized decision-making within a firm, because a supervisor can always use the sum of the risks of the units reporting to him as a conservative risk measure. But if risks are not sub-additive, adding them together gives an underestimate of combined risks, and this makes the sum of risks effectively useless as a risk measure. In risk management, it is desirable for risk estimates to be unbiased or if they are biased at least to be biased on the conservative side.

- If regulators use non-sub-additive risk measures, like they do VaR, to set capital requirements, a bank might be tempted to break itself up to reduce its regulatory capital requirements, because the sum of the capital requirements of the smaller units would be less than the capital requirement of the bank as a whole.
- Non-sub-additive risk measures can also inspire traders to break up their accounts, with separate accounts for separate risks, in order to reduce their margin requirements. This could be a matter of serious concern for the exchange because the margin requirements on the separate accounts would no longer cover the combined risks.

A very serious shortcoming of VaR is that it provides no handle on the extent of the losses that might be suffered beyond the threshold amount indicated by it. VaR is incapable of distinguishing between situations where losses in the tail are only a bit worse, and those where they are overwhelming. Indeed, VaR merely provides a lowest bound for losses in the tail of the loss distribution and has a bias toward optimism instead of the conservatism that ought to prevail in risk management. An alternative measure that does quantify the losses that might be encountered in the tail is the Conditional VaR (CVaR). Both VaR and CVaR require the user to a priori specify confidence level and holding period. While VaR represents a maximum loss one expects at a determined confidence level for a given holding period, CVaR is the loss one expects to suffer, provided that the loss is equal to or greater than VaR, that is (see Yamai, Y., Yoshiba, T., 2002a):

"CVaR is the expected value of the loss of the portfolio in the 100(1-cl)% worst cases during a holding period."

Figure 1: VaR and CVaR



Source: Adapted from: Yamai, Yoshiba, 2002a

Unconditional CVaR is defined as:

$$CVaR_{cl}(X) = E\left[X \mid X > VaR_{cl}(X)\right] = -cl^{-1}\int_{-\infty}^{VaR} xf(x)dx \tag{6}$$

and the conditional CVaR is given by:

$$CVaR^{t}_{cl,hp}(X) = E\left[\sum_{j=1}^{hp} X_{t+j} \mid \sum_{j=1}^{hp} X_{t+j} > VaR^{t}_{cl,hp}(X), \psi_{t}\right] \tag{7}$$

CVaR is very appealing as a risk measure in that it sums all values of x, weighted by $f(x)$, from minus infinity to VaR, thus taking into account of the sizes of losses beyond the VaR level. CVaR can be encountered in the academic literature under many names such as: Expected shortfall (ES), Expected tail loss (ETL), tail VaR, tail conditional expectation, mean excess loss, beyond VaR etc. CVaR measure has been used by insurance practitioners, especially casualty insurers for a long time as conditional average claim size. Reinsures are also familiar with conditional coverage of losses in excess of a threshold. For continuous loss distributions, CVaR at a given confidence level is the expected loss given that the loss is greater than the VaR at that level, or for that matter, the expected loss given that the loss is greater than or equal to the VaR. For distributions with possible discontinuities, however, it has a more subtle definition and can differ from either of those quantities, which for convenience in comparison can be designated by $CVaR^{+}$ and $CVaR^{-}$, respectively. $CVaR^{+}$ is also known as "mean shortfall", although the seemingly identical term "expected shortfall" has been interpreted in other ways in Acerbi, Nordio, Sirtori (2001), with the latter paper taking it as a synonym for CVaR itself), while "tail VaR" is a term that has been suggested for $CVaR^{-}$ (Artzner et al. 1999). Unlike $CVaR^{+}$ and $CVaR^{-}$, CVaR is a coherent measure of risk in the sense of Artzner et al. (1999). However, CVaR is no panacea and has its own flaws; Yamai and Yoshiba (2002b) find that both VaR and CVaR are not reliable during market turmoil and can give misleading results, although CVaR is a better choice than VaR.

With the dawn of a new "coherent" risk measure there exists a need for a new risk management paradigm. Risk management is primarily concerned with the risk of low-probability events that could result in catastrophic losses. Traditional VaR models tend to ignore extreme events and focus on modeling the entire empirical distribution of returns. By wrongly using the Central limit theorem it is often assumed that returns are normally or lognormally distributed, but little attention is paid to the distribution of the tails. The danger is that such risk models are prone to fail just when they are needed the most – in large market moves, when large losses occur. Estimation of the risks associated with rare events with limited data is inevitably problematic, and these difficulties increase as the events concerned become rarer. Inference about the extreme tails is always uncertain, because of low number of observations and sensitivity to the values of individual extreme observations. The key to estimating the distribution of extreme events is the extreme value theorem, which governs the distribution of extreme values, and shows how this distribution looks like in the limit, as the sample size increases.

The application of extreme value theory (EVT) to risk management was just the thing that the new "coherent" risk measure was missing to establish itself as a pretender to the throne currently held by "non-coherent" VaR. Estimation of the risks associated with rare events with limited data is inevitably problematic, and these difficulties increase as the events concerned become rarer. Inference about the extreme tail is always uncertain, because of low number of observations and sensitivity to the values of individual extreme observations. EVT models were primarily used in the field of civil engineering: engineers are required to design their structures to withstand the forces that might be reasonable to expect but are rarely experienced. Another standard field of application of EVT is hydrology, where engineers have long struggled with the question of how high dams, sea-walls and dikes should be to contain the probabilities of floods within reasonable limits. They have to do their calculations with even fewer observations than financial risk practitioners, and their quantile estimates are typically well out of the range of their sample data. EVT provides a framework in which an estimate of anticipated forces could be made using historical data. By definition, extreme events are rare, meaning that their estimates are often required for levels of a process that are greater that those in the available data set. This implies an extrapolation from observed levels to unobserved levels and extreme value theory provides a class of models to enable such extrapolation. In lieu of an empirical basis, asymptotic argument is used to generate EV models. Today EVT is used in traffic predictions in the telecommunications, alloy strength predictions, ocean wave modeling, thermodynamics of earthquakes, memory cell failure and many other fields. It is important to be aware of the limitations implied by the adoption of the extreme value paradigm. EV models are developed using asymptotic arguments, which should be kept in mind when applying them to finite samples. EV models are derived under idealized circumstances, which need not be true for a process being modeled.

Presuming *n* observations of P&L time series, if *X* is IID drawn from some unknown distribution $F(x) = P(X \leq x)$, estimating extreme value (EV) VaR/CVaR posses a significant problem because the distribution $F(x)$ is unknown. Help comes from Fisher-Tippett theorem (1928), which shows that as *n* gets large the distribution of tail of *X* converges to the Generalized extreme value distribution (GEV):

$$H_{\mu,\sigma,\xi}(x) = \left| \begin{array}{c} \exp\left(-\left[1+\xi(x-\mu)/\sigma\right]^{-1/\xi}\right) \\ \exp\left(-e^{-(x-\mu)/\sigma}\right) \end{array} \right. \quad \text{if} \quad \begin{array}{l} \xi \neq 0 \\ \xi = 0 \end{array} \tag{8}$$

where x satisfies the condition $1 + \xi(x-\mu)/\sigma > 0$. GEV distribution has three parameters: location parameter (μ), which is a measure of central tendency, scale parameter (σ), which is a measure of dispersion and tail index (ξ), which is a measure of the shape of the tail. GEV distribution has three special cases:

- If $\xi > 0$, GEV distribution becomes a Fréchet distribution, meaning that $F(x)$ is leptokurtotic.
- If $\xi = 0$, GEV distribution becomes a Gumbel distribution, meaning that $F(x)$ has normal kurtosis.
- If $\xi < 0$, GEV distribution becomes a Weibull distribution, meaning that $F(x)$ is platokurtotic, which is usually not the case with financial data.

Mean and variance are related to location and scale parameters of GEV distribution as follows (Dowd, 2002):

$$Mean = \mu + \left[\frac{\Gamma(1-\xi)-1}{\xi}\right]\sigma \text{ which converges to } \mu + 0{,}577216\sigma \text{ as } \xi \to 0 \qquad (9)$$

$$Variance = \left[\frac{\Gamma(1-2\xi)-\Gamma^2(1-\xi)}{\xi^2}\right]\sigma^2 \text{ which converges to } \frac{\pi^2}{6}\sigma^2 \text{ as } \xi \to 0 \qquad (10)$$

It is easy to obtain mean and variance from μ and σ, but one must be careful not to confuse the two since they differ significantly.

Quantiles of GEV distribution can be obtained by taking log of equation (8):

$$\log(cl) = \begin{cases} -\left(1+\xi(x-\mu)/\sigma\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ -\exp(-(x-\mu)/\sigma) & \xi = 0 \end{cases} \qquad (11)$$

Value of x is than calculated to get the quantiles or VaRs associated with a desired confidence level. EV VaR is calculated as (Dowd, 2002):

$$VaR_{cl} = \mu - \frac{\sigma}{\xi}\left[1-(-\log(cl))^{-\xi}\right] \quad \text{(Fréchet VaR, } \xi > 0) \qquad (12)$$

$$VaR_{cl} = \mu - \sigma \log\left[\log(1/cl)\right] \quad \text{(Gumbel VaR, } \xi = 0) \qquad (13)$$

There are no closed form CVaR formulas for Fréchet and Gumbel distributions but EV CVaR can be derived from EV VaR estimates using "average-tail VaR" algorithm set out in Dowd (2002). The fact that the CVaR is a probability weighted average of tail losses suggests that CVaR can be estimated as an average of tail VaRs. The approach suggested by Dowd (2002) to calculating CVaR is to divide the tail of the P&L distribution into a large number (n > 500) of equally distant slices, each of which has the same probability mass and then calculate the VaR for each slice. The mean of calculated tail VaRs gives CVaR. It is easily shown that CVaR is indeed estimable in a consistent way as the “average of 100cl% worst cases:

$$CVaR_{cl}^{(n)}(X) = -\frac{1}{\lfloor ncl \rfloor}\sum_{i=1}^{\lfloor ncl \rfloor} X_{i:n} \quad \text{where } X_{i:n} \text{ are order statistics} \tag{14}$$

$$CVaR_{cl}^{(n)}(X) \xrightarrow[n\to\infty]{} ETL_{cl}(X) \tag{15}$$

To estimate EV risk measures it is necessary to estimate EV parameters – μ, σ, and in the case of Fréchet distribution the tail index (ξ). The first two parameters μ and σ can be easily found using standard methods to obtain mean and standard deviation, and than transforming those through equations (12) and (13). Estimation of tail index is a bit more demanding. Embrechts et al. (1997) suggests determining the tail index of the distribution via Hill estimator:

$$\hat{\xi}_{n,k}^{(H)} = k^{-1}\sum_{j=1}^{k} \ln X_{j,n} - \ln X_{k+1,n} \tag{16}$$

where k, the tail threshold used in the Hill estimation has to be chosen arbitrarily, which is a major source of problems in practice. The Hill estimator is the average of the k most extreme observations, minus ($k+1$)th observation, which is next to the tail. There are two approaches to handling the trade off between bias and variance. The first approach, recommended by Embrechts et al. (1997), is based on estimating the Hill estimator for a range of k values and selecting the k values where the plot of the Hill estimator against k flattens out. Danielson, de Vries (1997) suggest finding an optimal value of k that minimizes MSE loss function and, in regards to MSE, reflects an optimal trade off between bias and variance. Their procedure takes a second-order approximation to the tail of the distribution and uses the fact that k is optimal (in the MSE sense) at the point where bias and variance reduce at the same rate. They suggest using a sub-sample bootstrapping procedure for finding the optimal value of k. Unfortunately, this approach, although brilliant, is impractical since it requires a very large sample size which is very difficult to obtain in practice. For this reason in this paper the approach recommended by Embrechts is adopted.

When forecasting CVaR, researchers mostly use either simple moving average models with GEV distribution or plain equally weighted historical models. I propose bootstrapping transformed tail losses, because it is shown that a similar approach when used for VaR estimation on volatile markets yields significant improvements over parametric and nonparametric approaches (see Žiković, 2007). CVaR models that are analyzed in this paper are: simple moving average volatility model (SMA) with Fréchet distribution, simple moving average volatility model (SMA) with

Gumbel distribution, bootstrapped historical simulation with 500 days observation period, GARCH volatility model with Fréchet distribution, GARCH volatility model with Gumbel distribution and bootstrapped HHS CVaR model.

CVaR for simple moving average volatility model (SMA) with Fréchet and Gumbel distribution and GARCH volatility model with Fréchet and Gumbel distribution, due to the lack of closed form solutions, are derived from their respective EV VaR estimates using "average-tail VaR" algorithm.

Historical simulation CVaR can be expressed as:

$$CVaR = E(X \mid X > VaR) = \left(\sum_{i=[ncl]}^{n} X_{n(i)} \right) / (n - [ncl]) \tag{17}$$

where $X_{n(1)} \leq X_{n(2)} \leq ... \leq X_{n(n)}$ are order statistics.

Parametric CVaR forecasts, even those based on GEV distribution should be very sensitive to the misspecification of the functional form of the losses and parameter estimates, especially tail index. Furthermore these models can not adequately or timely adapt to sudden changes in levels of volatility. Purely nonparametric CVaR estimation approaches, such as calculating CVaR from the untransformed historical data set of tail losses, are certain to be unreactive to sudden shifts in market regimes and occurrence of extreme events. This is exactly the same critique that applies to this approach when using them for VaR calculation. The logic shows that the weak points of risk measurement models cannot be ignored and they continually come back to haunt us even when we switch from one risk measure to another. The problems remain the same regardless whether we are estimating VaR or CVaR.

Hybrid historical simulation (HHS) model that was developed in Žiković (2007) and yielded excellent results for stock indexes from 12 EU new member states can be used as a basis for developing a semi-parametric approach to estimation of CVaR. I propose a Bootstrapped HHS CVaR model that starts by standardizing the tail losses in excess of HHS VaR by the latest GARCH volatility update for that point in time to form a series of standardized tail losses:

$$z_t = \frac{Tail \quad loss_t}{\sigma_t} \tag{18}$$

Since these standardized tail losses are now IID they are suitable for bootstrapping. Using bootstrapping new discrete PDFs of tail losses are derived which are than updated by the latest GARCH volatility forecasts:

$$\hat{F}_{(n)}(t) = F_{(n)}(t) \times \sigma_{t+1} \tag{19}$$

By taking the averages over a great number of volatility updated tail PDFs ($\hat{F}_{(n)}(t)$) CVaR forecasts are obtained. Besides being reactive to the latest market developments through the use of GARCH volatility updating the HHS CVaR approach also provides for an elegant way of calculating confidence intervals for CVaR estimates, based on bootstrapping that is free of any distributional assumptions. The only assumption that is made in the model is that the underlying data generating process can be described by a GARCH process. HHS CVaR model does not impose any distributional assumptions about the behavior of the tail losses, unlike EV CVaR models, and allows for the empirical distribution of tails to evolve over time.

Hybrid historical simulation (HHS) CVaR can be expressed as:

$$CVaR = E(X \mid X > VaR) = \left(\sum_{i=[ncl]}^{n} \hat{Z}_{n(i)} \right) / \left(n - [ncl]\right) \tag{20}$$

where $\hat{Z}_{n(1)} \leq \hat{Z}_{n(1)} \leq \ldots \leq \hat{Z}_{n(n)}$ are order statistics from volatility scaled bootstrapped series $\hat{Z}$.

The strong points and weaknesses of every model remain with them and that is way knowledge obtained in developing VaR models must not be wasted. Cutting-edge VaR estimation techniques can easily be adopted to serve a new "superior" risk measure – CVaR. Research in VaR estimation should by no means be discouraged, but instead intensified, because it could now serve a dual purpose – improving VaR estimates but also improving CVaR estimates.

A more robust series of statistical tests can be used when evaluating CVaR then VaR. Since CVaR forecasts the expected extreme losses it allows us to employ the standard best-fit statistics because the task is to measure the distance between realized and forecasted extreme losses. In order to statistically compare CVaR models, in the next section, each model will be graded by four symmetrical error statistics: the mean absolute error (MAE), two versions of the root mean squared error (RMSE), and the mean absolute percentage error (MAPE).

$$RMSE(1) = \sqrt{\frac{\sum_{i=1}^{T} \left| L_i^2 - CVaR_i^2 \right|}{T}} \tag{21}$$

$$RMSE(2) = \sqrt{\frac{\sum_{i=1}^{T} \left(\left|L_i\right| - \left|CVaR_i\right| \right)^2}{T}} \tag{22}$$

$$MAE = \frac{1}{T}\sum_{i=1}^{T}\left\| L_i \right| - \left| CVaR_i \right\| \qquad (23)$$

$$MAPE = \frac{1}{T}\sum_{i=1}^{T}\left| \frac{\left| L_i \right| - \left| CVaR_i \right|}{L_i} \right| \qquad (24)$$

Out of these error statistics, MAPE can be viewed as the most global of these statistics since it measures the deviation between forecasted and realized values in relative terms. By measuring the deviations in this manner MAPE statistic is the most comparable measure between different CVaR models. Furthermore, MAPE unlike RMSE does not square the deviations and in that manner does not put the most weight on the outliers, but redistributes it equally across all of the observations.

4. Data and results

The data used in the analyses of CVaR models are the daily log returns from stock indexes of countries that formed former Yugoslavia (Slovenia – SBI20 index, Croatia – CROBEX index, Serbia – BELEX index, Bosnia and Herzegovina (Republic of Srpska) – BIRS index, Montenegro – NEX20, Macedonia – MBI10), with the exception of Sarajevo stock exchange index SASE for which data for longer periods is not publicly available. The returns are collected for the period 01.01.2000 - 12.05.2008, which includes the latest market crisis in the global and regional markets. The calculated CVaR figures are for a one-day ahead horizon and a cut off level of 95 percent, i.e., the five percent of returns that fall into lower tail of the return distribution are considered to be extreme returns. To secure the same out-of-the-sample VaR backtesting period for all of the tested stock indexes, the out-of-the-sample data sets are formed by taking out 500 of the latest observations from each stock index where it was possible (SBI20, CROBEX and NEX20). For the rest of the indexes (BELEX, BIRS and MBI10) that have a short history, latest 250 observations are used. The rest of the observations are used as presample observations needed for CVaR starting values, tail index estimation required for EV approaches and volatility model calibration. Length of the tail losses data set used for backtesting depends on the number of VaR errors generated by each VaR model. The quality of CVaR forecasts does not only depend on CVaR estimation model but also on the quality of the VaR forecast. This can be easily demonstrated by the simple fact that a loss that might be extreme under one VaR model and as such is compared to the CVaR forecast might not exceed some other, more conservative VaR model.

All of the analyzed indexes show a strong positive mean, which is significantly different from zero, a finding that can be expected in emerging, fast growing stock markets. Distribution of returns is not symmetrical and shows significant positive asymmetry (except BIRS index). High excess kurtosis indicates the presence of extreme events that are unlikely to occur under the normality assumption. Consequently, all of the normality tests show that there is virtually no probability that the data generating processes behind these indexes are normally distributed. Ljung Box Q tests on mean adjusted returns and squared returns show that all analyzed stock indexes are characterized by significant autoregression and heteroskedasticity. GARCH and EGARCH representation of volatility with Student's T and GED distribution are used to capture the dynamics of data generating processes of analyzed indexes. The dynamics of the data generating processes are complex because changes in the efficiency of the market alter the long-run level and persistence of volatility. Furthermore, there is ample of empirical evidence on a positive relationship between trading volume and volatility. Supposing that some predictability (significant AR term) is present in the series, increasing efficiency tends to lower the level and persistence of volatility, but larger volume might push its level up. Volatility can be raised due to other reasons too, for example when news in the return series arrive more often and are of larger magnitude than usual (shift in the volatility of error term). The increasing integration of the local stock markets into the global capital market may only further amplify this effect. The descriptive statistics for the analyzed indexes are presented in table 1.

Table 1: Descriptive statistics for SBI20, CROBEX, BELEX, BIRS, NEX20 and MBI10 index in the period from the official beginning of the index until 12.05.2008.

	SBI20	CROBEX	BELEX	BIRS	NEX20	MBI10
Mean	0.0008	0.0008	0.0012	0.0007	0.0025	0.0022
Median	0.0006	0.0006	0.0009	0.0000	0.0001	0.0015
Minimum	-0.0635	-0.0903	-0.0538	-0.0737	-0.0756	-0.0713
Maximum	0.0831	0.1498	0.0657	0.0732	0.0967	0.0809
Standard deviation	0.0084	0.0139	0.0083	0.0140	0.0182	0.0171
Skewness	0.2585	0.5472	0.2251	-0.0042	0.5969	0.2125
Kurtosis	15.527	17.318	11.616	6.938	7.211	6.889
Normality tests:						
JB test (%)	0	0	0	0	0	0
Shapiro-Wilk/Francia (%)	0	0	0	0	0	0
Lilliefors test (%)	0	0	0	0	0	0
Autoregression	✓	✓	✓	✓	✓	✓
Heteroskedasticity	✓	✓	✓	✓	✓	✓

Source: Author's calculations

All of the characteristics that are found in the returns from the analyzed stock indexes indicate that classical VaR models could not forecast the true level of risk an investor would be faced with when investing in these markets. CVaR models in general, but especially those based on extreme value approach should adequately capture the risks since they a priori focus on the tail regions of the return distribution. Backtesting results for CVaR at 95% cut-off level are presented in tables 1-6 and figures 1-6 in the appendix.

For SBI20 index at 95% cut-off level according to all of the employed error statistics the Bootstrapped HHS CVaR approach was the best performing CVaR measure resulting in smallest deviations from realized tail losses, and minimizing the loss function. The worst performer was the GARCH extreme value (EV) approach based on Fréchet distribution with tail index equal to 0.3, resulting in the highest deviations from realized tail losses. For CROBEX index according to RMSE(1), RMSE(2) and MAE statistic the simple moving average (SMA) EV approach based on Gumbel distribution was the best performing CVaR model. According to MAPE statistic the GARCH EV approach with Gumbel distribution was the best performing CVaR model. The worst performers were the GARCH and the SMA EV approaches based on Fréchet distribution with tail index equal to 0.32. For BELEX index according to all of the employed error statistics the SMA EV approach based on Gumbel distribution was the best performing CVaR model. The worst performer was the SMA EV approach based on Fréchet distribution with tail index equal to 0.22. For BIRS index according to all of the employed error statistics the bootstrapped historical simulation (HS) with 500 days' window length was the best performing CVaR model. The worst performers were the GARCH and the SMA EV approaches based on Fréchet distribution with tail index equal to 0.275. For NEX20 index according to all of the employed error statistics the SMA EV approach based on Gumbel distribution was the best performing CVaR model. The worst performer was the SMA EV approach based on Fréchet distribution with tail index equal to 0.2, resulting in highest deviations from realized tail losses. For MBI10 index according to RMSE(1), RMSE(2) and MAE statistic the Bootstrapped HHS CVaR approach was the best performing CVaR model. According to MAPE statistic the SMA EV approach with Gumbel distribution was the best performing CVaR model. The worst performer was the GARCH EV approaches based on Fréchet distribution with tail index equal to 0.3.

As detailed in the previous section, due to its characteristics the MAPE statistic can be viewed as the most intuitive and reliable error statistic out of the selected ones. The ranking of tested CVaR models according to MAPE statistic is presented in table 2.

Table 2: Ranking of tested CVaR models according to MAPE statistic in the last 500 (250) days up until 12.05.2008.

CVaR models						
Score	1.	2.	3.	4.	5.	6.
SBI20	Bootstrap HHS	SMA EV (Gumbel)	GARCH EV (Gumbel)	Bootstrap HS 500	SMA EV (Frechet)	GARCH EV (Frechet)
CROBEX	GARCH EV (Gumbel)	SMA EV (Gumbel)	Bootstrap HS 500	Bootstrap HHS	GARCH EV (Frechet)	SMA EV (Frechet)
BELEX	SMA EV (Gumbel)	GARCH EV (Gumbel)	Bootstrap HHS	Bootstrap HS 500	GARCH EV (Frechet)	SMA EV (Frechet)
BIRS	Bootstrap HS 500	SMA EV (Gumbel)	Bootstrap HHS	GARCH EV (Gumbel)	SMA EV (Frechet)	GARCH EV (Frechet)
NEX20	SMA EV (Gumbel)	GARCH EV (Gumbel)	Bootstrap HS 500	Bootstrap HHS	GARCH EV (Frechet)	SMA EV (Frechet)
MBI10	SMA EV (Gumbel)	Bootstrap HHS	GARCH EV (Gumbel)	Bootstrap HS 500	SMA EV (Frechet)	GARCH EV (Frechet)

Source: Author's calculations

Overall the SMA EV approach based on Gumbel distribution was the best performing CVaR measure being the best ranked CVaR model for three out of six indexes (BELEX, NEX20 and MBI10). The SMA EV (Gumbel) model is followed by GARCH EV approach that also uses Gumbel distribution, Bootstrapped HHS CVaR and Bootstrapped HS500 CVaR model. The worst performers for all of the tested indexes are SMA and GARCH EV approach based on Fréchet distribution. These CVaR models based on Fréchet distribution greatly overestimated the expected averages of extreme (tail) losses.

Comparing the performance results of virtually the same models that use Gumbel instead of Fréchet distribution provides a clearer picture into the anatomy of the estimation problem. Models that used Gumbel distribution performed far better compared to models using Fréchet distribution. Since the models are the same but differ only in the choice of the underlying distribution this points to two possible reasons for such over-predictions of Fréchet distribution based CVaR models:

a) Tail indexes have been incorrectly calculated (they are too high) and/or

b) The use of GEV distributions with heavy tails in CVaR estimation provides excessively conservative estimates of average extreme (tail) losses.

Since estimation of tail index for CVaR models using Fréchet distribution was carried out across different time periods and the values remained virtually unchanged we can disregard the problem with estimation of tail indexes for now and conclude that for the tested stock indexes the use of GEV distributions with heavy tails in CVaR estimation provides overly conservative estimates of average tail losses. The performance of Bootstrapped HHS model is satisfactory, giving consistent estimates, with minimal deviations from realized extreme events, and being second only to approaches using Gumbel distribution.

The backtesting performance of tested CVaR models shows obvious consistency, with models using Gumbel distribution being always among top two models, and models using Fréchet distribution being always the worst performing CVaR models. These finding are somewhat surprising since similar CVaR models are ranked as the best and the worst performers, but clearly point to the importance of choosing the right extreme value distribution to describe the tails of the data, as well as accurate tail index estimation when using extreme value approach.

5. Conclusion

When forecasting CVaR measures researchers mostly use either simple moving average models with GEV distribution or plain equally weighted historical models. Parametric CVaR forecasts, even those based on GEV distribution should be very sensitive to the misspecification of the functional form of the losses and parameter estimates, especially tail index. Purely nonparametric CVaR estimation approaches, such as calculating CVaR from the untransformed historical data set of tail losses are certain to be unreactive to sudden shifts in market regimes and occurrence of extreme events. This is exactly the same critique that applies to these approaches when using them for VaR calculation. A new CVaR estimation approach named Bootstrapped Hybrid historical simulation CVaR is presented in the paper. The model is based on bootstrapping volatility transformed tail losses. Similar model has been shown to yield significant improvements over parametric and nonparametric approaches when used as a VaR model on volatile and illiquid markets.

The analyzed stock indexes from Slovenia, Croatia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia significantly differ in statistical characteristics from the developed markets. Distribution of returns on these indexes is not symmetrical and shows significant positive asymmetry (except BIRS index). High excess kurtosis indicates the presence of extreme events that are unlikely to occur under the normality assumption. Furthermore, all of the analyzed stock indexes are characterized by significant autoregression, which is not common in the developed stock markets. These statistical characteristics make CVaR estimation more challenging and require sophisticated treatment of mean and volatility data generating processes.

The backtesting performance of tested CVaR models in the regional stock markets shows obvious consistency, with Gumbel distribution based CVaR models being always among top two CVaR models, and Fréchet distribution based models being always the worst performing CVaR models. These findings are somewhat surprising since similar CVaR models are ranked as the best and the worst performers, but clearly point to the importance of choosing the right extreme value distribution to describe the tails of the data, as well as accurate tail index estimation when using extreme value approach. It can be concluded that for the tested stock indexes the use of GEV distributions with heavy tails in CVaR estimation provides overly conservative estimates of average tail losses. The newly proposed Bootstrapped HHS model performed satisfactory, giving consistent estimates, with minimal deviations from realized extreme events, and being second only to approaches that use Gumbel distribution. The quality of CVaR forecasts does not only depend on CVaR estimation model but also on the quality of the VaR forecasts, since superior VaR models yield a smaller number of tail events, which in turn eases the CVaR estimation. Having this in mind the focus of future research should be on improving both VaR and CVaR estimation techniques as well as finding superior combinations of VaR/CVaR models, because information from such combination of risk measures can serve as a solid basis for decision making by investors.

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Kvantificiranje ekstremnih rizika na burzama: Analiza na primjeru država u sastavu bivše Jugoslavije[1]

Saša Žiković[2]

Sažetak

Jedan od razloga zbog kojeg su investitori bili nespremni na visoke gubitke na tržištima kapitala koji su se dogodili nakon početka hipotekarne krize u SAD-u zasigurno se nalazi u neobičnoj činjenici da su mnogi investitori bili uvjereni da se zbog velikog broja neovisnih agenata na tržištima kapitala, ona moraju ponašati prema teoremu centralnog limita tj. da su povrati na tržištima kapitala normalno distribuirani. Očito je da je paradigma normalnosti još jedanput iznevjerila, a ugled mjera rizika koje se temelje na VaR-u ozbiljno je uzdrman. Alternativna mjera rizika koja u ovim teškim vremenima puno obećaje i uspješno kvantificira ekstremne gubitke jest kondicionalni VaR (CVaR). VaR predstavlja gubitak koji se može ostvariti od određene investicije, u promatranom razdoblju, uz određenu vjerojatnost, dok je CVaR prosječni gubitak koji se može očekivati ukoliko je realizirani gubitak veći ili jednak predviđenom VaR-u. U ovom radu testiranje CVaR modela je provedeno na burzovnim indeksima Slovenije, Hrvatske, Bosne i Hercegovine, Srbije, Crne Gore i Makedonije. Provedeno testiranje pokazuje da su CVaR modeli uspješni u predviđanju ekstremnih gubitaka koji su se dogodili na analiziranim tržištima. Posebno uspješnim su se pokazali modeli temeljeni na generaliziranoj distribuciji ekstremnih vrijednosti te predloženi CVaR model temeljen na hibridnoj povijesnoj simulaciji.

Ključne riječi: *ekstremni gubitci, kondicionalni VaR, teorija ekstremnih vrijednosti, hibridna povijesna simulacija*

JEL klasifikacija*: G24, C14, C22, C52, C53*

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Appendices

Table 1: Backtesting results for CVaR forecasts (SBI20 index, ξ = 0.3, cl = 0.95, period 26.4.2006 - 12.5.2008)

Cl	**95%**					
SBI20	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.0458	0.0216	**0.0182**	0.0376	0.0198	0.0185
RMSE2	0.0283	0.0099	**0.0081**	0.0225	0.0090	0.0100
MAE	0.0240	0.0075	**0.0059**	0.0203	0.0070	0.0068
MAPE	1.1153	0.3287	**0.2467**	1.0464	0.2994	0.3354

Source: Author's calculations

Table 2: Backtesting results for CVaR forecasts (CROBEX index, ξ = 0.32, cl = 0.95, period 11.5.2006 - 12.5.2008)

Cl	**95%**					
CROBEX	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.0466	0.0218	0.0221	0.0402	**0.0197**	0.0198
RMSE2	0.0282	0.0090	0.0096	0.0251	**0.0086**	0.0099
MAE	0.0261	0.0074	0.0084	0.0235	**0.0071**	0.0073
MAPE	1.1167	**0.2877**	0.3790	1.2295	0.3226	0.3241

Source: Author's calculations

Table 3: Backtesting results for CVaR forecasts (BELEX index, ξ = 0.22, cl = 0.95, period 15.5.2007 - 9.5.2008)

Cl	**95%**					
BELEX	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.0337	0.0169	0.0179	0.0412	**0.0154**	0.0158
RMSE2	0.0216	0.0084	0.0086	0.0258	**0.0062**	0.0079
MAE	0.0167	0.0057	0.0069	0.0255	**0.0055**	0.0071
MAPE	1.1147	0.3478	0.4945	1.3828	**0.3239**	0.5880

Source: Author's calculations

Table 4: Backtesting results for CVaR forecasts (BIRS index, ξ = 0.275, cl = 0.95, period 4.5.2007 - 12.5.2008)

Cl	**95%**					
BIRS	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.0676	0.0251	0.0208	0.0738	0.0242	**0.0203**
RMSE2	0.0463	0.0112	0.0082	0.0496	0.0099	**0.0080**
MAE	0.0436	0.0095	0.0070	0.0488	0.0085	**0.0066**
MAPE	1.7705	0.3943	0.2985	1.6498	0.2890	**0.2312**

Source: Author's calculations

Table 5: Backtesting results for CVaR forecasts (NEX20 index, ξ = 0.2, cl = 0.95, period 25.4.2006 - 12.5.2008)

Cl	**95%**					
NEX20	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.0683	0.0319	0.0371	0.0686	**0.0289**	0.0345
RMSE2	0.0434	0.0137	0.0198	0.0434	**0.0118**	0.0172
MAE	0.0382	0.0115	0.0176	0.0412	**0.0104**	0.0155
MAPE	1.2029	0.3495	0.7216	1.3046	**0.3178**	0.5451

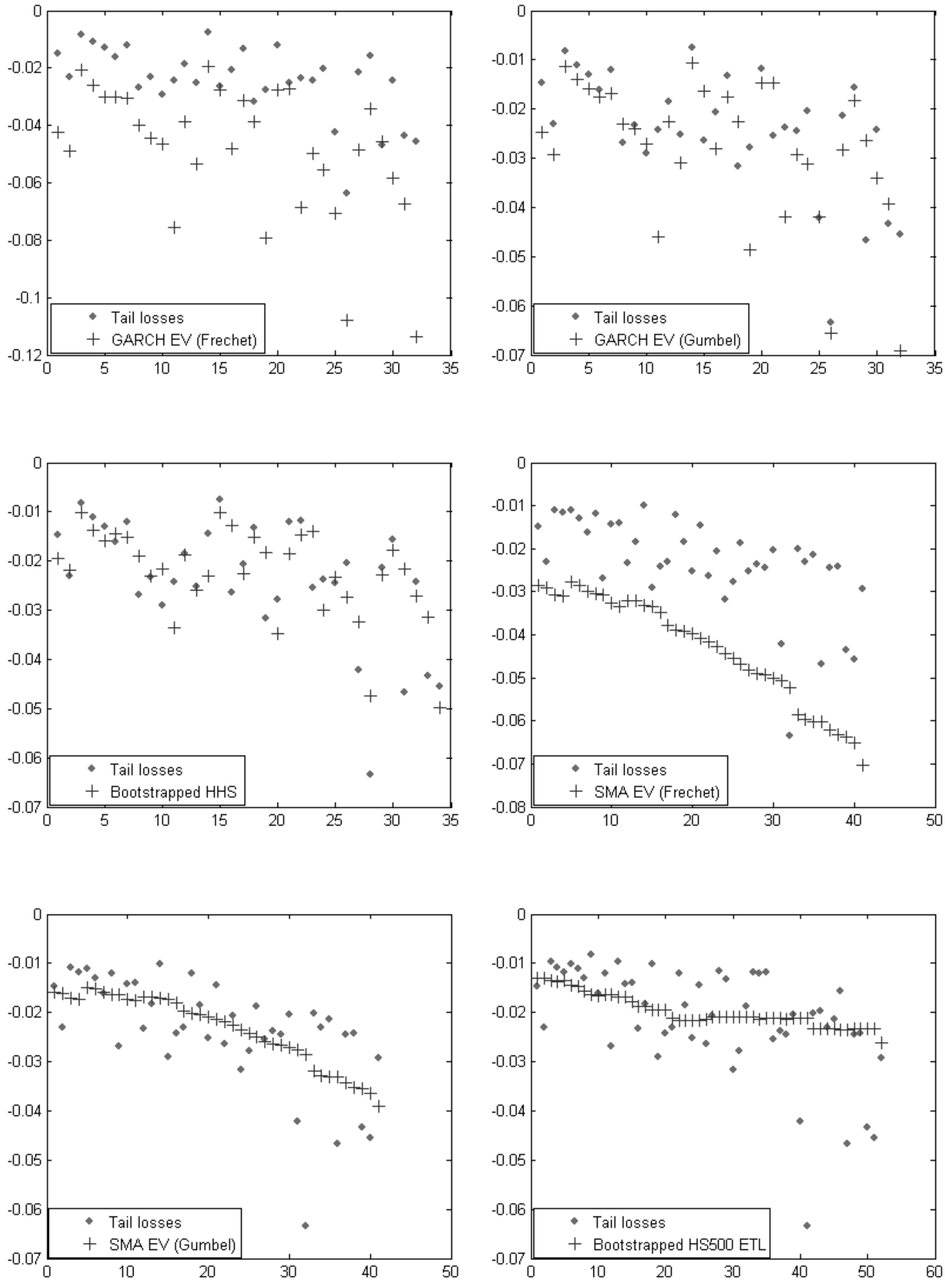
Source: Author's calculations

Table 6: Backtesting results for CVaR forecasts (MBI10 index, ξ = 0.3, cl = 0.95, period 11.5.2007 - 12.5.2008)

Cl	**95%**					
MBI10	GARCH EV (Frechet)	GARCH EV (Gumbel)	Bootstrap HHS	SMA EV (Frechet)	SMA EV (Gumbel)	Bootstrap HS 500
RMSE1	0.1029	0.0368	**0.0278**	0.0902	0.0313	0.0296
RMSE2	0.0709	0.0164	**0.0121**	0.0620	0.0134	0.0144
MAE	0.0686	0.0145	**0.0106**	0.0606	0.0120	0.0126
MAPE	1.9332	0.4246	0.3979	1.9089	**0.3639**	0.4743

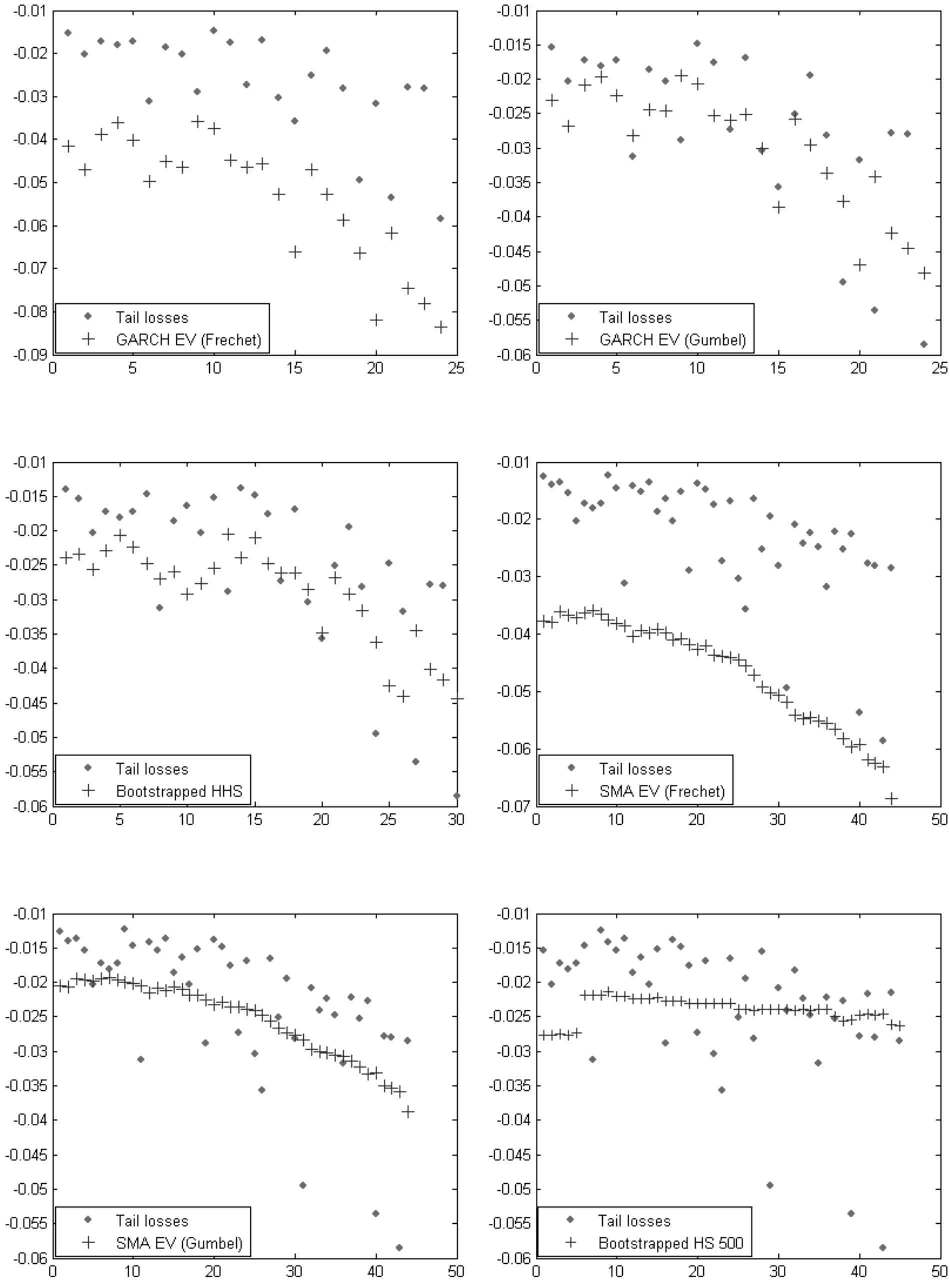
Source: Author's calculations

Figure 1: Tail losses and CVaR forecasts for SBI20 index in the period 26.4.2006 - 12.5.2008 (cl = 0.95, ξ = 0.3)



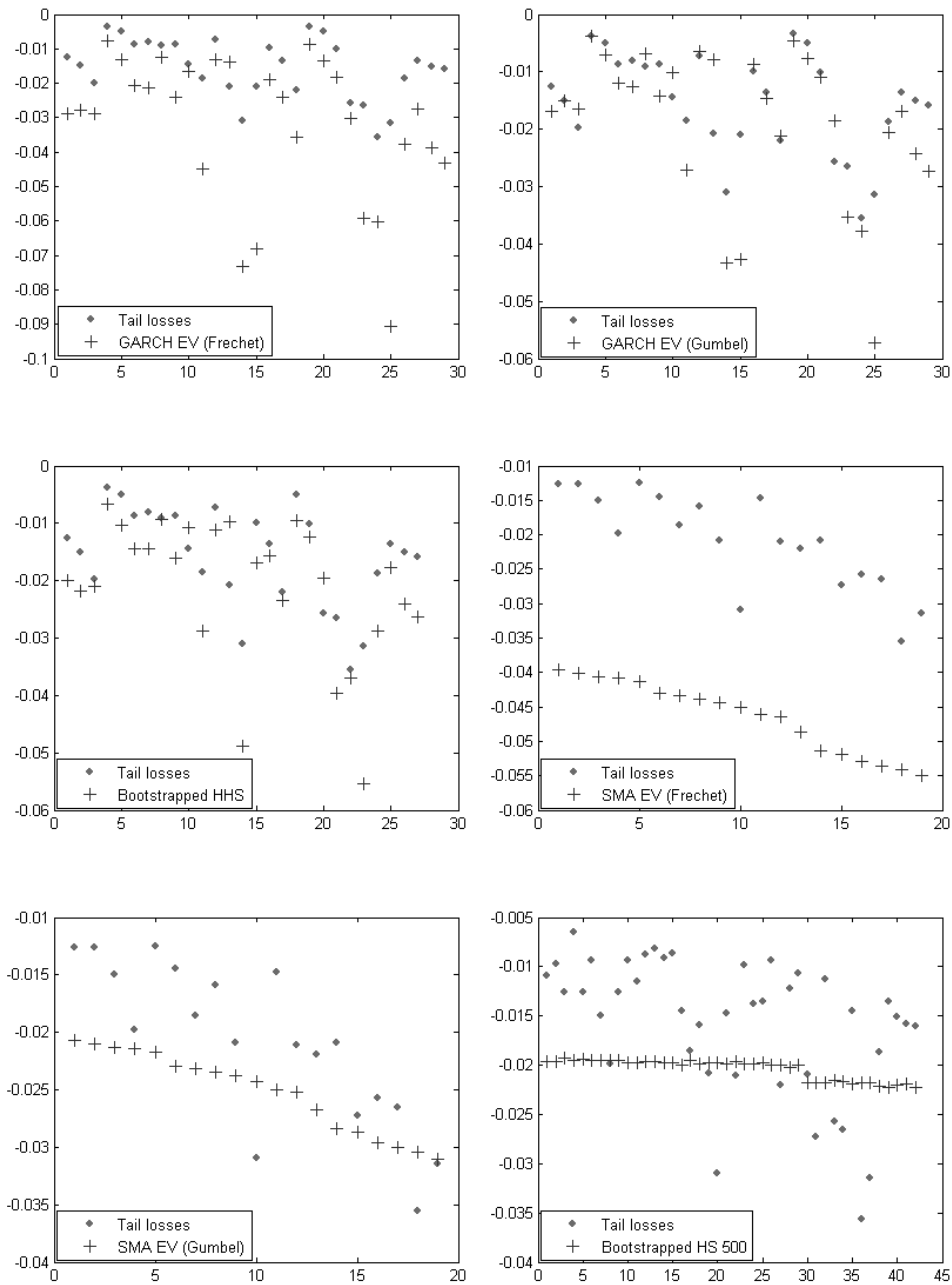
Source: Author's calculations

Figure 2: Tail losses and CVaR forecasts for CROBEX index in the period 11.5.2006 - 12.5.2008 (cl = 0.95, ξ = 0.32)



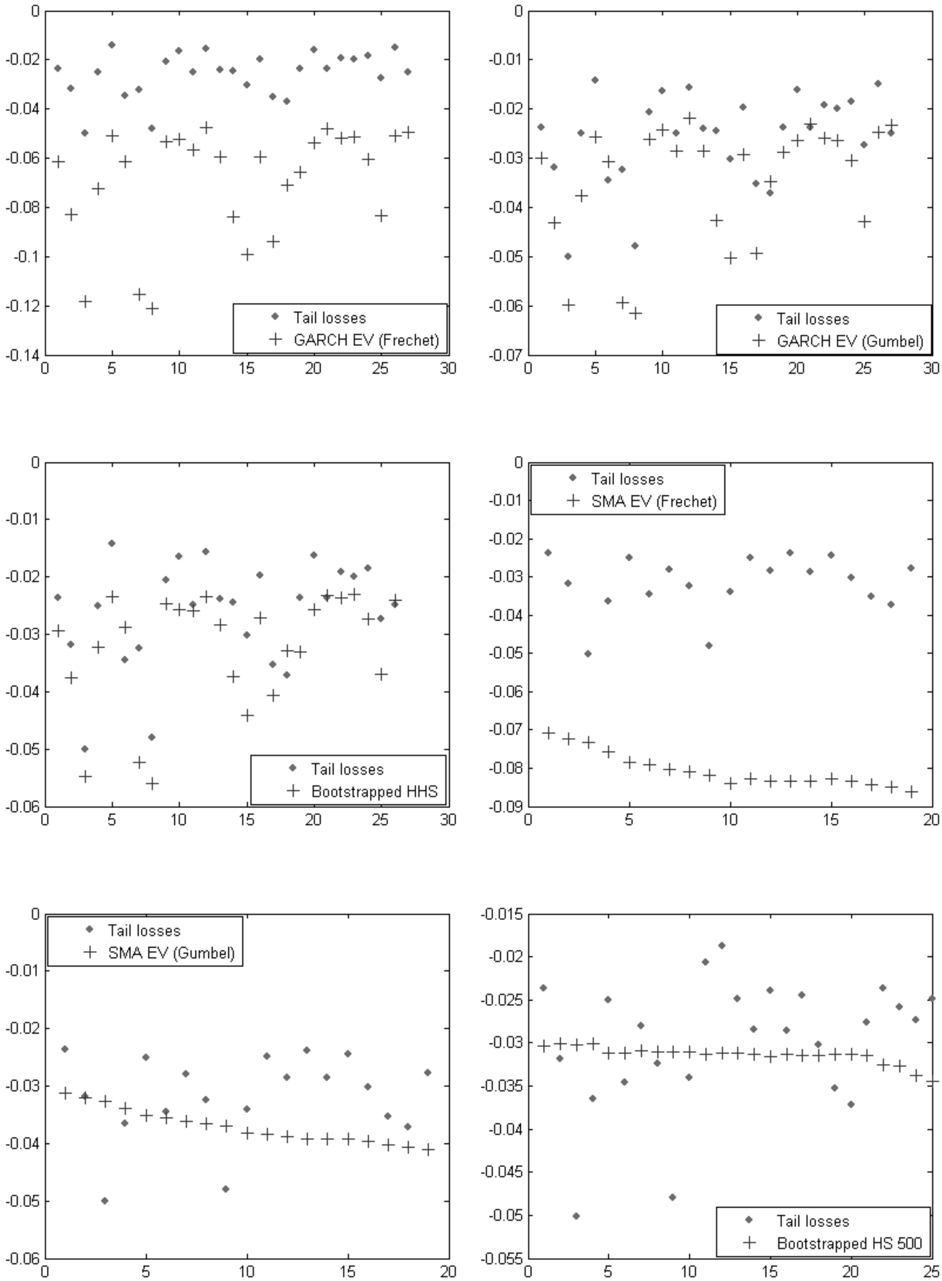
Source: Author's calculations

Figure 3: Tail losses and CVaR forecasts for BELEX index in the period 15.5.2007 - 9.5.2008 (cl = 0.95, ξ = 0.22)



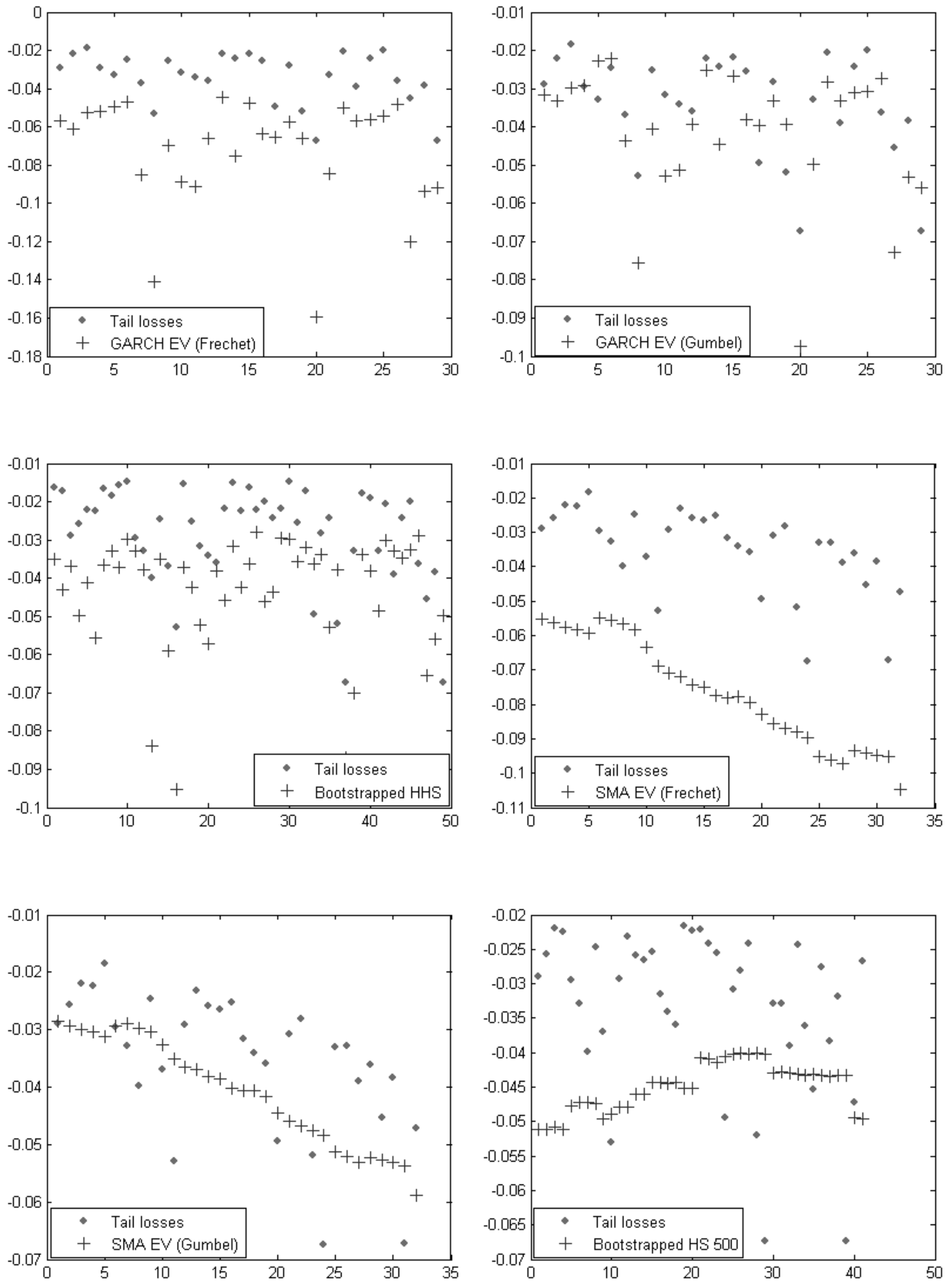
Source: Author's calculations

Figure 4: Tail losses and CVaR forecasts for BIRS index in the period 4.5.2007 - 12.5.2008 (cl = 0.95, ξ = 0.275)



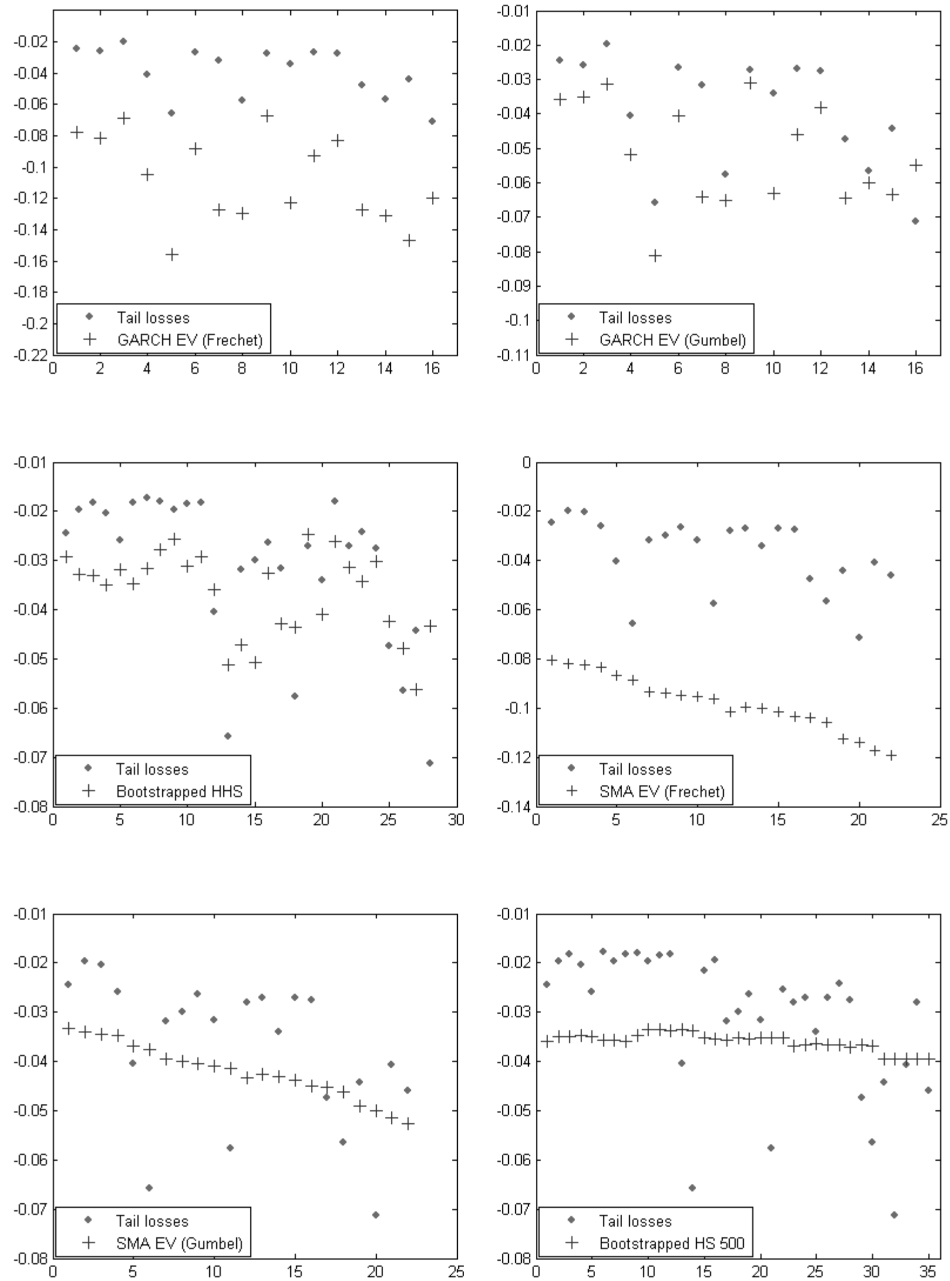
Source: Author's calculations

Figure 5: Tail losses and CVaR forecasts for NEX20 index in the period 25.4.2006 - 12.5.2008 (cl = 0.95, ξ = 0.2)



Source: Author's calculations

Figure 6: Tail losses and CVaR forecasts for MBI10 index in the period 11.5.2007 - 12.5.2008 (cl = 0.95, ξ = 0.3)



Source: Author's calculations